Philippine International Mathematical Olympiad

PhIMO

MOCK PAPER 2020

JUNIOR SECONDARY

NAME OF PARTICIPANT: _____ **DATE**: _____

SCHOOL NAME: _____

GENERAL INSTRUCTIONS:

1. Do not open the booklet until you are told to do so.
2. You are given 90 minutes to attempt all 25 questions.
3. Ensure to enter the necessary information asked in the Answer Sheet such as your name, participant number, country, and year level.
4. Record your answers neatly on the Answer Sheet provided.
5. Marks are awarded for correct answers only. There is no penalty for incorrect answers.
6. Calculators are not allowed.
7. All figures are not drawn to scale. They are intended only as aids.
8. Start answering when the proctor gives the signal.

Part 1 (Questions 1 to 10):

There are 10 multiple-choice questions. Choose the best answer from the four possible choices

Each question carries 2 marks

Part 2 (Questions 11 to 25):

There are 20 open-ended questions, each requiring a single answer. Write your answer on the box provided in the Answer Sheet

Questions 11 to 20, each carries 3 marks

Questions 21 to 25, each carries 5 marks

DO NOT REMOVE this exam paper from the exam venue

Part 1: 1st to 10th Multiple Choice Questions

1. In a certain football league, the only way to score is to kick a field goal for 3 points or a score a tonch down for 7 points. Thus the scores 1, 4 and 8 are not possible. How many positive scores are not possible?
   A. 5 B. 6 C. 9 D. 11 E. 13

2. What is the number of distinct real numbers x which have the property that the median of the five numbers $x, 6, 4, 1, 9$ is equal to their mean?
   A. 0 B. 1 C. 2 D. 3 E. 5

3. In a group of five friends, the sum of the ages of each group of four of them are 124, 128, 130, 136, and 142. What is the age of the youngest of the friends?
   A. 18 B. 21 C. 23 D. 25 E. 34

4. Eleven teams play in a soccer tournament. Each team must play each of the other teams exactly once. If a game ends in a tie, each team gets 1 point. For the games that do not end in a tie, the winning team gets 5 points and the losing team gets 0 points.

   Which of the following is a possible value for the total number of points earned by the 11 teams by the end of the tournament?
   A. 92 B. 196 C. 257 D. 290 E. 450

5. If $x = \log_9 2$ and $y = \log_5 4$, find $\log_6 15$ in terms of x and y.
   A. $\dfrac{2x+y}{xy}$ B. $\dfrac{2x+y}{y(1+4x)}$ C. $\dfrac{4x+y}{y(1+2x)}$ D. $\dfrac{y+4x}{2}$ E. $\dfrac{x(4x+y)}{y+1}$

6. Given that $P(x)$ is a polynomial such that $P(x^2+1) = x^4+5x^2+3$, what is $P(x^2-1)$?
   A. x^4+x^2-3 B. x^4+5x^2-1 C. $x^2(x+1)(x-1)$ D. x^4+x^2+1
   E. x^4-x^2-1

7. Which number below is the greatest?
   A. 6^{100} B. 5^{200} C. 4^{300} D. 3^{400} E. 2^{500}

8. A cube measuring 100 units on each side is painted only on the outside and cut into unit cubes. The number of cubes with paint only on two sides is
   A. 1000 B. 1125 C. 1176 D. 980 E. none of these

9. What is the length of the shortest path that begins at the point (2,5), touches the x-axis and then ends at a point on the circle

$$(x+6)^2 + (y-10)^2 = 16$$

   A. 12 B. 13 C. $4\sqrt{10}$ D. $6\sqrt{5}$ E. $4+\sqrt{89}$

10. Wilson's Theorem states that if n is a prime number, then n divides $(n-1)!+1$. Which of the following is a divisor of $12! \cdot 6! + 12! + 6! + 1$?
    A. 21 B. 77 C. 91 D. 115 E. 143

Part 2: 11^{th} to 30^{th} Open-ended Questions

11. What is the smallest prime divisor of $5^{20^{20}} + 1$.

12. A bowl contains 100 piece of colored candy: 28 green, 20 red, 12 yellow, 10 blue, 20 brown, and 10 orange. If you are blindflolded as you pick and eat candy from this bowl, then how many pieces must you eat in order to guarantee that you have eaten at least 15 of the same color?

13. Nine nonnegative numbers have average 10. What is the greatest possible value for their median?

14. Given that the mean, median, range and the only mode of 80 integers are also 80. If A is the largest integer among those 80 integers, find the maximum value of A.

15. There are 100 students who want to sign up for the class Introduction to Acting. There are three class sections for Introduction to Acting, each of which will fit exactly 20 students. The 10 students including Dendi and Nico, are put in a lottery, and 60 of them are randomly selected to fill up the classes. What is the probability that Dendi and Nico end up getting into the same section for the class?

16. Determine the exact value of $\sqrt{2020(2021)(2022)(2023)+1}$.

17. Find all values of $\dfrac{y}{x}$ where $x^2 - 6xy + 8y^2 = 0$, and $x \neq 0$.

18. Let x be a real number with the property that $x + \dfrac{1}{x} = 7$. Suppose $S_m = x^m + \dfrac{1}{x^m}$, determine the value of S_4.

19. The sum of 3 real numbers is known to be zero. If the sum of their cubes is e^{π}, what is their product equal to?

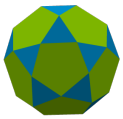
20. Given that 7,999,999,999 has at most two prime factors, find its largest prime factor.

21. How many positive integers x are there such that $3x$ has 3 digits and $4x$ has four digits?

22. An icosidodecahedron is a convex polyhedron with 20 triangular faces and 12 pentagonal faces. How many vertices does it have (see next page for figure)?

23. The surface area of a right rectangular prism (a box) is 48 square feet, and the sum of its length, width, and height is 13 feet. What is the length of the longest diagonal connecting two corners of the box?

24. What is the area of the largest circle contained in an equilateral triangle of area $8\sqrt{3}$.



Question 22

25. A circle of radius 3 crosses the center of a square of side length. Find the positive difference between the area of the nonoverlapping portions of the figures.