PhIMO

MOCK PAPER 2020

SECONDARY 3

NAME OF PARTICIPANT: _____ **DATE**: _____

SCHOOL NAME: _____

GENERAL INSTRUCTIONS:

1. Do not open the booklet until you are told to do so.
2. You are given 90 minutes to attempt all 25 questions.
3. Ensure to enter the necessary information asked in the Answer Sheet such as your name, participant number, country, and year level.
4. Record your answers neatly on the Answer Sheet provided.
5. Marks are awarded for correct answers only. There is no penalty for incorrect answers.
6. Calculators are not allowed.
7. All figures are not drawn to scale. They are intended only as aids.
8. Start answering when the proctor gives the signal.

Part 1 (Questions 1 to 10):
   There are 10 multiple-choice questions. Choose the best answer from the four possible choices
   Each question carries 2 marks

Part 2 (Questions 11 to 25):
   There are 20 open-ended questions, each requiring a single answer. Write your answer on the box provided in the Answer Sheet
   Questions 11 to 20, each carries 3 marks
   Questions 21 to 25, each carries 5 marks

Part 1: 1st to 10th Multiple Choice

1. On a certain test, the average score for the women in the class is 83, while the average score for the men in the class is 71. If the average score of all the stuents in the class is 80, then what percentage of the students are women?
   A. 60%   B. 65%   C. 70%   D. 75%   E. 80%

2. An odd integer between 600 and 800 is divisible by 7 and also divisible by 9. What is the sum of its digits?
   A. 27   B. 21   C. 18   D. 12   E. 7

3. If $9^{-x} = 7$, then what is the value of 27^{2x+1}?
   A. $\dfrac{27}{7\sqrt{7}}$ B. $189\sqrt{7}$ C. $\dfrac{343}{27}$ D. $\dfrac{7\sqrt{7}}{27}$ E. $\dfrac{27}{343}$

4. If $x@y = \dfrac{x^3 - y^3}{x - y}$, for how many real values of x does $x@1 = 0$?
   A. 0   B. 1   C. 2   D. 3   E. 4

5. If the squares of two consecutive odd numbers differ by 1000, what is the sum of the two odd numbers?
   A. 500   B. 520   C. 540   D. 530   E. 540

6. Find the number of integers n such that $\dfrac{5n + 26}{2n + 3}$ is an integer.
   A. 1   B. 2   C. 3   D. 4   E. 5

7. Now is December. Which month will it be after 43^{689} months?
   A. April   B. May   C. June   D. July   E. August
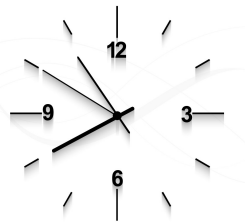
Question 8

8. What is the measure of the acute angle between the hour and minute hands of a correctly working clock at 4:18?
   A. 12°   B. 15°   C. 18°   D. 21°   E. 24°

1
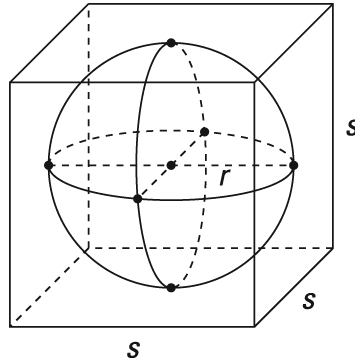
Question 9

9. If a unit sphere $(r = 1)$ circumscribes a cube, what is the surface area of the said cube?
   A. 8 sq. units   B. 6 sq. units   C. 4 sq. units   D. 2 sq. units   E. 1 sq. unit

10. Find all ordered pairs (x, y) such that $(x - 2y)^2 + (y - 1)^2 = 0$.
    A. (1,2)   B. (-1,2)   C. (-2,1)   D. (2,1)   E. (2,-1)

Part 2: 11th to 30th Open-ended Questions

11. There are n people in a room. They shake hands with each other once. If they shake hands 120 times, find n.

12. Carl and Bob can demolish a building in 6 days, Anne and Bob can do it in 3, Anne and Carl can do it in 5. How many days does it take all of them working together if Carl gets injured at the end of the first day and can't come back? Express your answer as a fraction in lowest terms.

13. The average of some numbers is 90. After adding 6 numbers, 125, 216, 343, 512, 729, 1000, the average of these numbers went high to 105. Find the sum of original numbers.

14. Jethro goes northeast for $10\sqrt{2}$ km, then goes east for 12km, goes west for 8km and goes southwest for $15\sqrt{2}$km. How far is he now from the original position?

15. Given that r and s are relatively prime positive integers such that $\dfrac{r}{s} = \dfrac{2(\sqrt{2} + \sqrt{10})}{5(\sqrt{3} + \sqrt{5})}$, find $r + s$.

16. Determine the exact value of $\sqrt[3]{2 + \sqrt{5}} + \sqrt[3]{2 - \sqrt{5}}$.

17. Find $x - y$, given that $x^4 = y^4 + 24$, $x^2 + y^2 = 6$, and $x + y = 3$.

18. When 29 is added to a number, it becomes a perfect square. When 100 is added to the same number, it becomes a perfect fourth power. What is the number?

19. Find the number of positive integers less than 2020 that are divisible by 16 or 20.

20. If $A = 10^9 - 987654321$ and $B = \dfrac{123456789 + 1}{10}$, what is the value of $\sqrt{AB}$?
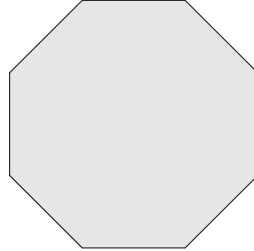
Question 21

21. Find the interior angle between two sides of a regular octagon (in degrees).

22. A rectangle has perimeter 10 and diagonal $\sqrt{15}$. What is its area?

23. A rectangle has sides of length $\sin x$ and $\cos x$ for some x. What is the largest possible area of such rectangle?

24. What is the exact measure of each angle(in degrees) of a regular polygon if it has 819 diagonals?

Question 25

25. If two fair dice are tossed, what is the probability that their sum is divisible by 5?