Philippine International Mathematical Olympiad

PhIMO

MOCK PAPER 2020

SENIOR SECONDARY

NAME OF PARTICIPANT: _____ **DATE:** _____

SCHOOL NAME: _____

GENERAL INSTRUCTIONS:

1. Do not open the booklet until you are told to do so.
2. You are given 90 minutes to attempt all 25 questions.
3. Ensure to enter the necessary information asked in the Answer Sheet such as your name, participant number, country, and year level.
4. Record your answers neatly on the Answer Sheet provided.
5. Marks are awarded for correct answers only. There is no penalty for incorrect answers.
6. Calculators are not allowed.
7. All figures are not drawn to scale. They are intended only as aids.
8. Start answering when the proctor gives the signal.

Part 1 (Questions 1 to 10):

There are 10 multiple-choice questions. Choose the best answer from the four possible choices

Each question carries 2 marks

Part 2 (Questions 11 to 25):

There are 20 open-ended questions, each requiring a single answer. Write your answer on the box provided in the Answer Sheet

Questions 11 to 20, each carries 3 marks

Questions 21 to 25, each carries 5 marks

DO NOT REMOVE this exam paper from the exam venue

Part 1: 1st to 10th Multiple Choice Questions

1. Which of the following is the largest?
   A. $2^{\log_5 6}$ B. $2^{\log_6 5}$ C. $3^{\log_6 5}$ D. $3^{\log_5 6}$ E. 3

2. Let A be the set of points in the $xy-$plane satisfying the equation $|x| + |y| = 7$. The area of A is equal to
   A. 7 B. 28 C. 42 D. 63 E. 98

3. What is the smallest altitude in the triangle with sides $20, 21$, and 29.
   A. 21 B. $\dfrac{21 \cdot 20}{29}$ C. $\dfrac{20 \cdot 29}{23}$ D. $\dfrac{20 \cdot 29}{21}$ E. 20

4. If $a + b + c = a^2 + b^2 + c^2 = a^3 + b^3 + c^3 = \frac{3}{2}$, then abc equals
   A. -2 B. $-\dfrac{1}{16}$ C. 10 D. $-\dfrac{1}{2}$ E. 20

5. Find the exact value of $\sqrt{14^3 + 15^3 + 16^3 + \ldots + 24^3 + 25^3}$.
   A. 224 B. 104 C. 312 D. 336 E. 676

6. How many triples (a, b, c) are solutions of the system of two equations

$$a + b = 2, \quad ab - c^2 = 1?$$

   A. 0 B. 1 C. 2 D. 3 E. infinitely many

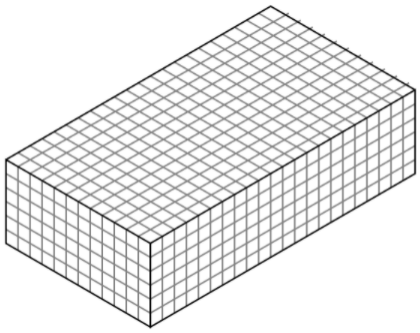
7. Suppose $f(x)$ is a function that satisfies the equation $5f\left(\dfrac{1}{x}\right) + f(x) - x = 3$ for all non-zero real numbers x. Determine the value of $f(4)$.
   A. $\dfrac{37}{96}$ B. $\dfrac{27}{53}$ C. 21 D. 12 E. $\dfrac{15}{4}$

8. Let a, b, c, d be positive integers such that $\log_a b = \dfrac{3}{2}$ and $\log_c d = \dfrac{5}{4}$. If it is known that $a - c = 9$, determine the value of $b - d$.
   A. 47 B. 87 C. 93 D. 105 E. 64

9. Find the number of ordered pairs (x, y), where x is an integer and y is a perfect square that satisfies the equation $y + 4907 = (x - 90)^2$.
   A. 0 B. 2 C. 4 D. 6 E. 8

10. Find the exact value of $\sin^4 \dfrac{\pi}{8} + \sin^4 \dfrac{3\pi}{8} + \sin^4 \dfrac{5\pi}{8} + \sin^4 \dfrac{7\pi}{8}$.
    A. 1 B. 1.5 C. 2 D. 2.5 E. 3.5

Part 2: 11^{th} to 30^{th} Open-ended Questions



Question 17

11. Kier has a salary of $\$202,020,020$ this year, Next year, his salary will increase by 300%. The year after, his salary will decrease by 75%. Compute his salary in two years.

12. Let $f(x) = x^4$ and $g(x) = \dfrac{1}{x^4}$. Find the value of $f''(2)g''(2)$.

13. Compute the smallest root of $x^4 - x^3 - 5x^2 + 2x + 6$.

14. Find all real x that satisfy the equation $\sqrt[3]{20x + \sqrt[3]{20x + 20}} = 20$.

15. Compute $\lim_{x\to 3} \dfrac{x^2 + 4x - 21}{x^2 - 4x + 3}$.

16. In ΔABC it is known that $AB = 13$, $BC = 14$ and $AC = 15$. Let D and E be the feet if the altitudes from A and B, respectively. Find the circumference of the circumcircle of ΔCDE.

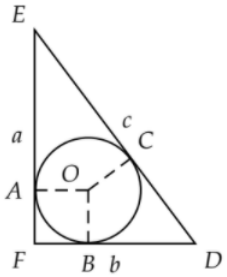
17. A 6 cm $\times$ 12 cm $\times$ 22 cm rectangular block of wood is painted blue and then cut into small cubes, each of which ahs a surface area of 6 cm^2. Find the number of small cubes that have blue paint on exactly two faces.

18. Given a regular hexagon $ABCDEF$, compute the probability that a randomly chosen point inside the hexagon is inside triangle PQR, where P is a midpoint of AB, Q is the midpoint of CD, and R is the midpoint of EF.

19. A regular dodecagon is inscribed in a circle of radius 10. Find its area.

20. Find the minimum value of $xy + xz + yz$ given that x, y, z are real numbers that satisfies the equation $x^2 + y^2 + z^2 = 1$.

21. How many ways are there to put 7 identical oranges into 4 identical packages so that each package has at least one apple?



Question 25

22. Determine the exact value of

$$\frac{2}{\dfrac{1}{\sqrt{2}+\sqrt[4]{8}+2}+\dfrac{1}{\sqrt{2}+\sqrt[4]{8}-2}}$$

23. Find the integer which is closests to $\dfrac{(1+\sqrt{3})^4}{4}$.

24. Find the 2021th−smallest x with $x > 1$, that satisfies the following relation:

$$\sin(\ln x) + 2\cos(3\ln x)\sin(2\ln x) = 0.$$

25. A circle is inscribed in a right triangle with sides a, b, and c, where c is the hypotenuse, as shown in the diagram. Find the radius of the circle using the sides of the triangle.